

ELASTICITY AND PLASTICITY

**Introduction.
Mathematical foundations (tensors)**

Prof. Mieczysław Kuczma

ISE PUT

Structural Engineering

Organizational issues

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Contact:

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- Email: mieczyslaw.kuczma@put.poznan.pl

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Diploma thesis: Elasticity and Plasticity, Concrete Constructions,
(Topic of MSc thesis may be agreed).

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- any book on elasticity and plasticity.

Elasticity, Plasticity, Rheology

Theory of Elasticity (TE)

is a part of continuum mechanics. It is concerned with movements and deformations of elastic bodies, i.e. such ones that after removal of external loadings (actions) the bodies return to their original shape. **Elastic deformations are reversible.** **FILM** from own laboratory experiments.

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Theory of Plasticity (TP)

is a generalization of TE. It is concerned with the states of a material above yield point (plastic stress), where after unloading some **permanent (plastic) deformations** remain. Application of TP allows one to make use of reserves of strength in structures. **FILM** from own laboratory tests.

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Theory of Plasticity (TP)

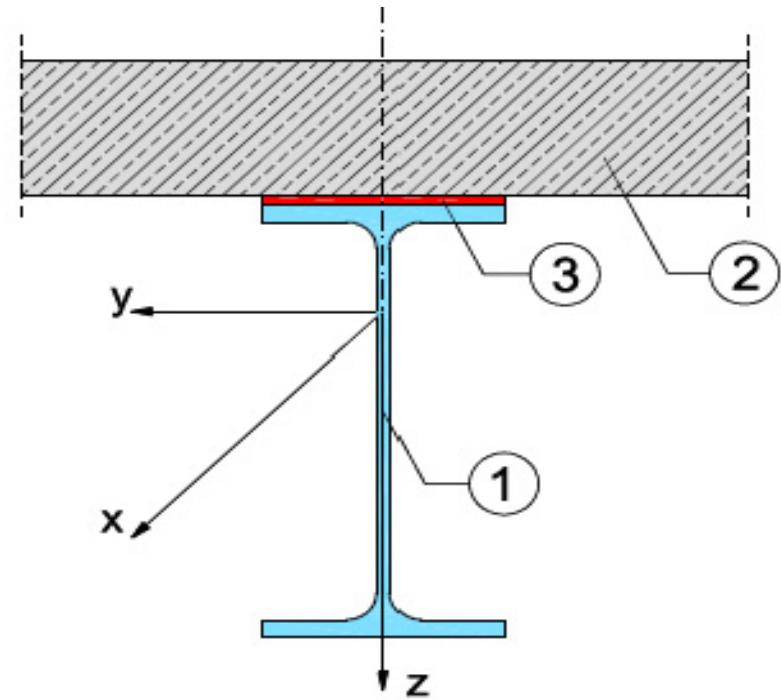
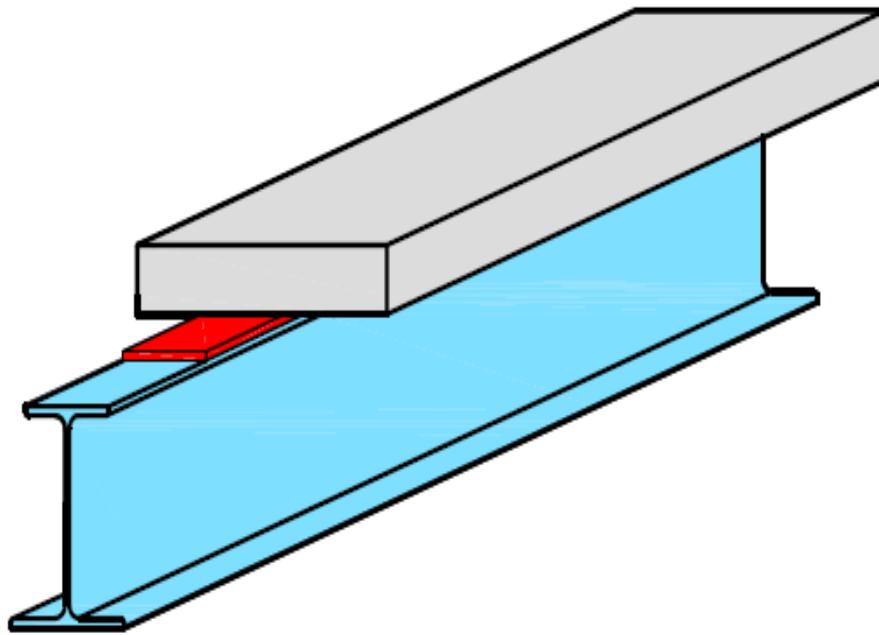
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Rheology

is a generalization of both TE & TP. It takes into account the influence of time manifested in creep, relaxation of stress, rates of deformations.

Visco-elasticity.

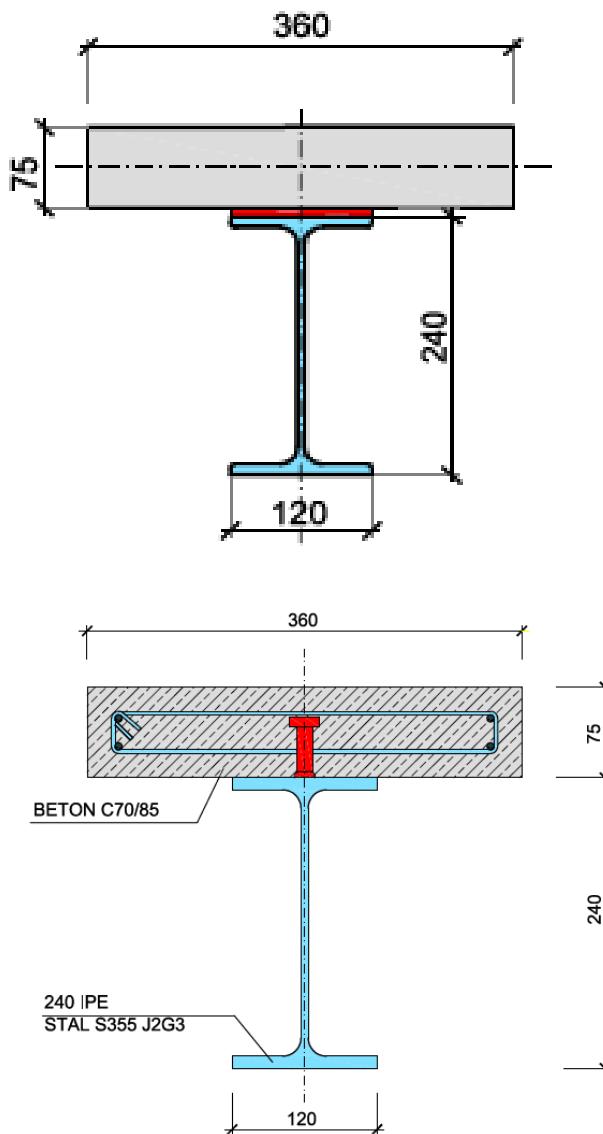
Formulation of the problem



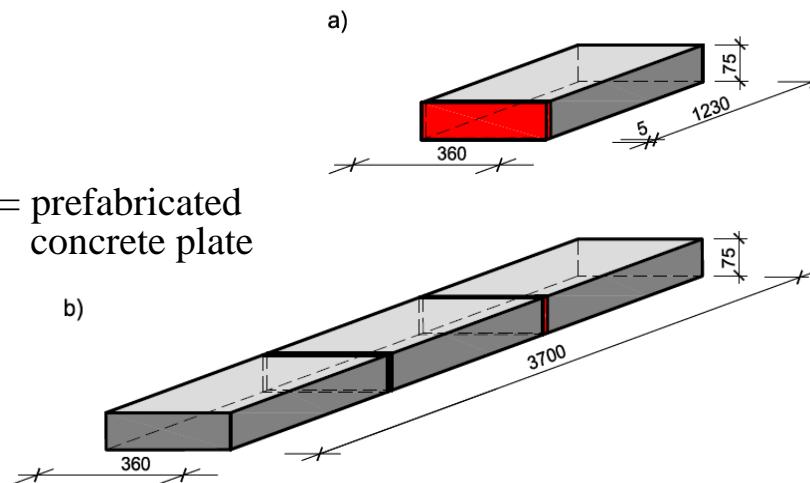
- 1 - steel girder
- 2 - concrete plate (slab)
- 3 - connection layer (interface bond)

Composite T-beam and its cross-section

• Tested beams



pcp = prefabricated concrete plate



- B1** – pcp rests freely on steel girder via cilindrical rollers with diameter of 5 mm
- B2** – pcp bonded with steel girder by flexible adhesive SikaTack®-Panel, $g_{adh} = 6 \pm 2$ mm
- B3** – pcp bonded with steel girder by stiff adhesive Sikadur®-30 Normal, $g_{adh} = 3$ mm
- B4** – as beam B3, but $g_{adh} = 5$ mm
- B5** – monolithic concrete plate joint with steel girder by steel shear studs 1/2" (13 mm), length 50 mm on cicrle foot bonded by Sikadur®-30 Normal
- B6** – monolithic concrete plate joint with steel girder by steel shear studs 1/2" (13 mm), length 50 mm



Experimental investigations using:

- **Instron 8804** - testing machine
(two frames and two actuators ± 500 kN)
- **Aramis & Pontos** (touchless measurements)
- **Traveller & sensors, tensometers (wire gauges)**

Tensors

Scalars

Physical quantities having magnitude only are represented by scalars (numbers), e.g. temperature T , density ρ . **Scalars are tensors of order zero** (in physical space: $1 = 3^0$). **Scalars are independent of the coordinate system.**

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Physical quantities which are defined by both magnitude and direction are called vectors, e.g. displacement u_i , force F_i . **Vectors possess 3 components and are called tensors of order one** ($3 = 3^1$).
Components of vectors depend on the coordinate system.

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Tensors (of order two)

Physical quantities that describe e.g. state of stress σ_{ij} , state of strain ε_{ij} are called tensors. Tensors of order 2 have 9 components ($3^2 = 9$).
Components of tensors depend on the coordinate system.