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Macierz sztywności prostokątnego ortotropowego elementu płyty

Oznaczenia:
a - połowa długości elementu na osi x, b - połowa długości elementu na osi y
Numery lokalne i współrzędne węzłów: $1\,(-a,-b)$ $2\,(a,-b)$ $3\,(a,b)$ $4\,(-a,b)$

$$\mathbf{D} = \begin{bmatrix} D_x & D_1 & 0 \\ D_1 & D_y & 0 \\ 0 & 0 & D_{xy} \end{bmatrix}$$

$$\mathbf{Kq} = \mathbf{P} - \mathbf{F}_0$$

$$\begin{aligned}
K_{1,1} &= 1/a^3/b^3*(14*D_{xy}*a^2*b^2+10*D_x*b^4+10*D_y*a^4+5*b^2*a^2*D_1)/10 \\
K_{2,1} &= 1/a/b^2*(2*b^2*D_{xy}+5*b^2*D_1+10*D_y*a^2)/10 \\
K_{2,2} &= 4/15/a/b*(5*D_y*a^2+2*b^2*D_{xy}) \\
K_{3,1} &= -1/a^2/b*(2*a^2*D_{xy}+10*D_x*b^2+5*a^2*D_1)/10 \\
K_{3,2} &= -D_1 \\
K_{3,3} &= 4/15/a/b*(5*D_x*b^2+2*a^2*D_{xy}) \\
K_{4,1} &= 1/a^3/b^3*(-14*D_{xy}*a^2*b^2-10*D_x*b^4+5*D_y*a^4-5*b^2*a^2*D_1)/10 \\
K_{4,2} &= 1/a/b^2*(-2*b^2*D_{xy}-5*b^2*D_1+5*D_y*a^2)/10 \\
K_{4,3} &= 1/a^2/b*(a^2*D_{xy}+5*D_x*b^2)/5 \\
K_{4,4} &= 1/a^3/b^3*(14*D_{xy}*a^2*b^2+10*D_x*b^4+10*D_y*a^4+5*b^2*a^2*D_1)/10 \\
K_{5,1} &= 1/a/b^2*(-2*b^2*D_{xy}-5*b^2*D_1+5*D_y*a^2)/10 \\
K_{5,2} &= 2/15/a/b*(5*D_y*a^2-4*b^2*D_{xy}) \\
K_{5,3} &= 0 \\
K_{5,4} &= 1/a/b^2*(2*b^2*D_{xy}+5*b^2*D_1+10*D_y*a^2)/10 \\
K_{5,5} &= 4/15/a/b*(5*D_y*a^2+2*b^2*D_{xy}) \\
K_{6,1} &= -1/a^2/b*(a^2*D_{xy}+5*D_x*b^2)/5 \\
K_{6,2} &= 0 \\
K_{6,3} &= -2/15/a/b*(-5*D_x*b^2+a^2*D_{xy}) \\
K_{6,4} &= 1/a^2/b*(2*a^2*D_{xy}+10*D_x*b^2+5*a^2*D_1)/10 \\
K_{6,5} &= D_1 \\
K_{6,6} &= 4/15/a/b*(5*D_x*b^2+2*a^2*D_{xy}) \\
K_{7,1} &= -1/a^3/b^3*(-14*D_{xy}*a^2*b^2+5*D_x*b^4+5*D_y*a^4-5*b^2*a^2*D_1)/10 \\
K_{7,2} &= -1/a/b^2*(-2*b^2*D_{xy}+5*D_y*a^2)/10 \\
K_{7,3} &= -1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2)/10 \\
K_{7,4} &= -1/a^3/b^3*(14*D_{xy}*a^2*b^2-5*D_x*b^4+10*D_y*a^4+5*b^2*a^2*D_1)/10 \\
K_{7,5} &= -1/a/b^2*(b^2*D_{xy}+5*D_y*a^2)/5 \\
K_{7,6} &= -1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2+5*a^2*D_1)/10 \\
K_{7,7} &= 1/a^3/b^3*(14*D_{xy}*a^2*b^2+10*D_x*b^4+10*D_y*a^4+5*b^2*a^2*D_1)/10 \\
K_{8,1} &= 1/a/b^2*(-2*b^2*D_{xy}+5*D_y*a^2)/10 \\
K_{8,2} &= 1/a/b*(5*D_y*a^2+2*b^2*D_{xy})/15 \\
K_{8,3} &= 0 \\
K_{8,4} &= 1/a/b^2*(b^2*D_{xy}+5*D_y*a^2)/5 \\
K_{8,5} &= 2/15/a/b*(5*D_y*a^2-b^2*D_{xy}) \\
K_{8,6} &= 0 \\
K_{8,7} &= -1/a/b^2*(2*b^2*D_{xy}+5*b^2*D_1+10*D_y*a^2)/10 \\
K_{8,8} &= 4/15/a/b*(5*D_y*a^2+2*b^2*D_{xy})
\end{aligned}$$

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\begin{aligned}
K_{9,1} &= 1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2)/10\\
K_{9,2} &= 0\\
K_{9,3} &= 1/a/b*(5*D_x*b^2+2*a^2*D_{xy})/15\\
K_{9,4} &= -1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2+5*a^2*D_1)/10\\
K_{9,5} &= 0\\
K_{9,6} &= -2/15/a/b*(-5*D_x*b^2+4*a^2*D_{xy})\\
K_{9,7} &= 1/a^2/b*(2*a^2*D_{xy}+10*D_x*b^2+5*a^2*D_1)/10\\
K_{9,8} &= -D_1\\
K_{9,9} &= 4/15/a/b*(5*D_x*b^2+2*a^2*D_{xy})\\
K_{10,1} &= -1/a^3/b^3*(14*D_{xy}*a^2*b^2-5*D_x*b^4+10*D_y*a^4+5*b^2*a^2*D_1)/10\\
K_{10,2} &= -1/a/b^2*(b^2*D_{xy}+5*D_y*a^2)/5\\
K_{10,3} &= 1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2+5*a^2*D_1)/10\\
K_{10,4} &= -1/a^3/b^3*(-14*D_{xy}*a^2*b^2+5*D_x*b^4+5*D_y*a^4-5*b^2*a^2*D_1)/10\\
K_{10,5} &= -1/a/b^2*(-2*b^2*D_{xy}+5*D_y*a^2)/10\\
K_{10,6} &= 1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2)/10\\
K_{10,7} &= 1/a^3/b^3*(-14*D_{xy}*a^2*b^2-10*D_x*b^4+5*D_y*a^4-5*b^2*a^2*D_1)/10\\
K_{10,8} &= -1/a/b^2*(-2*b^2*D_{xy}-5*b^2*D_1+5*D_y*a^2)/10\\
K_{10,9} &= -1/a^2/b*(a^2*D_{xy}+5*D_x*b^2)/5\\
K_{10,10} &= 1/a^3/b^3*(14*D_{xy}*a^2*b^2+10*D_x*b^4+10*D_y*a^4+5*b^2*a^2*D_1)/10\\
K_{11,1} &= 1/a/b^2*(b^2*D_{xy}+5*D_y*a^2)/5\\
K_{11,2} &= 2/15/a/b*(5*D_y*a^2-b^2*D_{xy})\\
K_{11,3} &= 0\\
K_{11,4} &= 1/a/b^2*(-2*b^2*D_{xy}+5*D_y*a^2)/10\\
K_{11,5} &= 1/a/b*(5*D_y*a^2+2*b^2*D_{xy})/15\\
K_{11,6} &= 0\\
K_{11,7} &= -1/a/b^2*(-2*b^2*D_{xy}-5*b^2*D_1+5*D_y*a^2)/10\\
K_{11,8} &= 2/15/a/b*(5*D_y*a^2-4*b^2*D_{xy})\\
K_{11,9} &= 0\\
K_{11,10} &= -1/a/b^2*(2*b^2*D_{xy}+5*b^2*D_1+10*D_y*a^2)/10\\
K_{11,11} &= 4/15/a/b*(5*D_y*a^2+2*b^2*D_{xy})\\
K_{12,1} &= 1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2+5*a^2*D_1)/10\\
K_{12,2} &= 0\\
K_{12,3} &= -2/15/a/b*(-5*D_x*b^2+4*a^2*D_{xy})\\
K_{12,4} &= -1/a^2/b*(2*a^2*D_{xy}-5*D_x*b^2)/10\\
K_{12,5} &= 0\\
K_{12,6} &= 1/a/b*(5*D_x*b^2+2*a^2*D_{xy})/15\\
K_{12,7} &= 1/a^2/b*(a^2*D_{xy}+5*D_x*b^2)/5\\
K_{12,8} &= 0\\
K_{12,9} &= -2/15/a/b*(-5*D_x*b^2+a^2*D_{xy})\\
K_{12,10} &= -1/a^2/b*(2*a^2*D_{xy}+10*D_x*b^2+5*a^2*D_1)/10\\
K_{12,11} &= D_1\\
K_{12,12} &= 4/15/a/b*(5*D_x*b^2+2*a^2*D_{xy})
\end{aligned}
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Wektor sił przywęzłowych od obciążenia równomiernego $q_0 = \text{const}$.

$$\mathbf{F}_q^e = -\int_{\Omega^e} \mathbf{N}^T q_0 = -q_0 ab\,(1,\, b/3,\, -a/3,\, 1,\, b/3,\, a/3,\, 1,\, -b/3,\, a/3,\, 1,\, -b/3,\, -a/3)^T$$

$$
\begin{aligned}
F_1 &= -q_0 * a * b \\
F_2 &= -q_0 * a * b^2/3 \\
F_3 &= q_0 * a^2 * b/3 \\
F_4 &= -q_0 * a * b \\
F_5 &= -q_0 * a * b^2/3 \\
F_6 &= -q_0 * a^2 * b/3 \\
F_7 &= -q_0 * a * b \\
F_8 &= q_0 * a * b^2/3 \\
F_9 &= -q_0 * a^2 * b/3 \\
F_{10} &= -q_0 * a * b \\
F_{11} &= q_0 * a * b^2/3 \\
F_{12} &= q_0 * a^2 * b/3
\end{aligned}
$$